DOCUMENT RESUME

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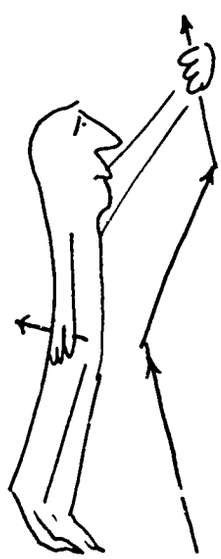
ABSTRACT

This is the second of a series of three programmed instruction booklets on vectors developed by Harvard Project Physics. It covers adding two or more vectors together, and finding a third vector that could be added to two given vectors to make a sum of zero. For other booklets in this series, see SE 015 549 and SE 015 551. (DT)

Project Physics **Programmed Instruction**

Vectors 2



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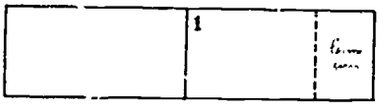
Vectors 2 Adding Vectors

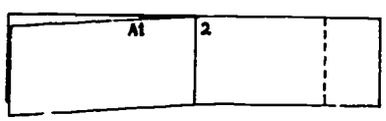
Adding vectors is an important technique for you to understand and be able to use. After going through this set of programmed materials you will be able to add two or more vectors together and obtain the resultant vector. The next three sample questions represent the kinds of questions you should be able to answer after you have finished Vectors 2. If you can already answer these frames, you need not take Vectors 2. In that case you can go on to Vectors 3.

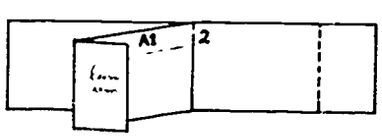
INSTRUCTIONS

1. Frames: Each frame contains a question. Answer the question by writing in the blank space next to the frame. Frames are numbered 1, 2, 3, ...

2. Answer Blacks: To find an answer to a frame, turn the page. Answer blacks are numbered A1, A2, A3, ... This booklet is designed so that you can compare your answer with the given answer by folding back the page, like this:







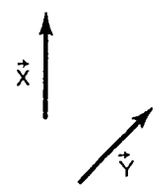
3. Always write your answer before you look at the given answer.

4. If you get the right answers to the sample questions, you do not have to complete the program.

Sample Question A

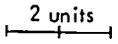
Answer Space

Given are two vectors, $\vec{X}$ and $\vec{Y}$, represented by the arrows drawn here.



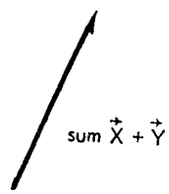
(i) Draw an arrow to represent the vector sum (resultant).

(ii) Give its magnitude.



Answer to A

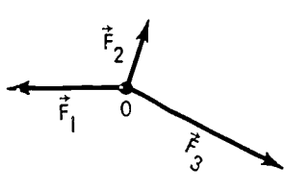
(i)



(ii) 3.7 units

Sample Question B

Three forces acting on an object, O, can be represented by arrows as drawn below. What is the resultant force on the object, that is, what is the vector sum of the three forces?



Answer Space

Answer to B

Resultant

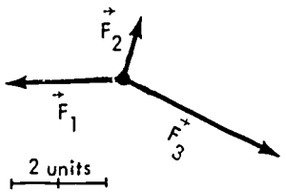
Resultant Force, F_R shown.

Sample Question C

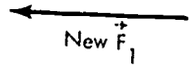
Answer Space

Forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ (from the last frame) are shown acting on a car. You found the resultant force by adding these vectors together tip-to-tail as shown at the left.

What should the magnitude of $\vec{F}_1$ have been if you wanted the resultant force to be zero?



Answer to C



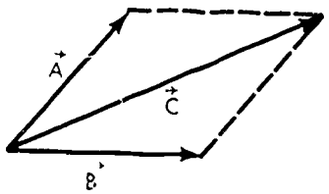
$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

if the magnitude of $\vec{F}_1$ is 3.5 units.

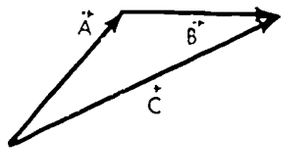
The Parallelogram Law

A vector is an entity having both magnitude and direction; vectors add by the parallelogram law as shown here, where $\vec{A}$ and $\vec{B}$ represent two vector quantities.

It can be drawn either



or



The vector sum of $\vec{A} + \vec{B}$ is $\vec{C}$ and can be drawn in two ways. Both ways of drawing the parallelogram law shown above are equivalent, but the "tip-to-tail" method on the right will be shown to be the more powerful since it can be extended easily to more than two vectors.

There are many physical quantities which have both direction and magnitude and add together according to the parallelogram law. In Part I of the vectors program the displacement vector was introduced, and Part II will begin with the addition of displacements.

1

Read the panel on the opposite page.

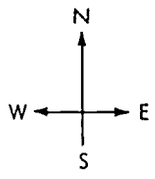
You learned in Part I of the program that a vector quantity has both magnitude and direction.

What other property will a vector quantity have?

A1

Vector quantities add according to the parallelogram law.

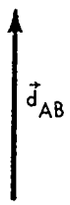
2



Let us use vectors to represent a trip around the city block. The first leg of the trip starts at intersection A, and is represented by $\vec{d}_{AB}$, the displacement vector drawn from A to B.

(i) What is the magnitude of the vector $\vec{d}_{AB}$?

(ii) What is its direction?



Scale:
1 cm = 100 m

A2

(i) 245 meters (approx.)

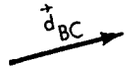
(ii) north

3

On the panel draw the second leg of the trip around the block, namely from B to C.

(i) Give the direction and magnitude of the displacement vector $\vec{d}_{BC}$.

(ii) Give the total distance traveled on the first two legs of this trip



(i) a few degrees North of East
170 meters

(ii) 415 meters
(A to B = 245 m, B to C = 170 m)

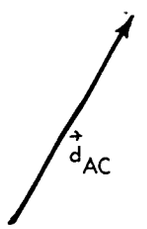
4

Draw the vector $\vec{d}_{AC}$ between points A and C. (This goes diagonally across the block.)

(i) Give the magnitude and direction of $\vec{d}_{AC}$.

(ii) What is the difference (in meters) between the distance traveled from points A to B to C, and the magnitude of the vector $\vec{d}_{AC}$?

A4



(i) 330 m
a few degrees North of Northeast

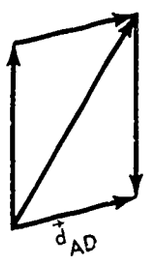
(ii) 85 m Difference

5

The displacement vector from A to C, $\vec{d}_{AC}$, is the resultant of adding $\vec{d}_{AB}$ and $\vec{d}_{BC}$.

The displacement vector $\vec{d}_{AD}$ is the resultant of adding $\vec{d}_{AC}$ and (i)_____.

(ii) What is the resultant of $\vec{d}_{BC}$ and $\vec{d}_{CD}$?



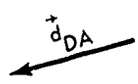
(i) $\vec{d}_{CD}$

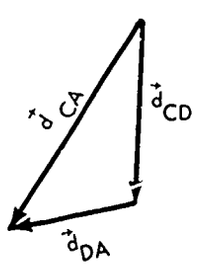
(ii) $\vec{d}_{BD}$

6

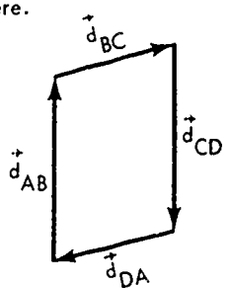
The final leg of the trip around the block, from intersection D to A, is given by the displacement vector $\vec{d}_{DA}$.

Draw the vector sum of $\vec{d}_{CD}$ and $\vec{d}_{DA}$.




$\vec{d}_{CA}$
$\vec{d}_{CD}$
$\vec{d}_{DA}$

The four legs of the trip around the block can be represented by the four separate vectors shown here.



Whot is the sum of these four vectors?

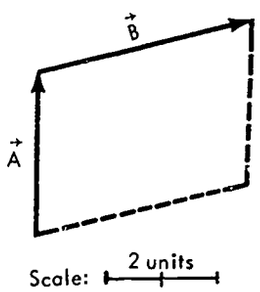
zero.

8

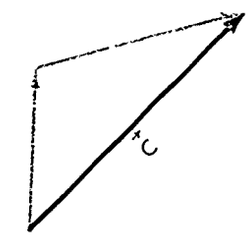
If the vector $\vec{C}$ is the sum of vectors $\vec{A}$ and $\vec{B}$, we can write: $\vec{A} + \vec{B} = \vec{C}$.

(i) Given $\vec{A}$ and $\vec{B}$ as shown, draw the vector sum $\vec{C}$.

(ii) Find the direction and magnitude of $\vec{C}$ by measuring the scale drawing.



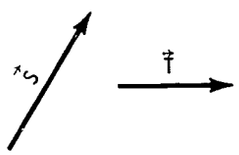
(i)



(ii) direction: 43° from $\vec{A}$.
magnitude: 5.2 units.

Two arrows representing the vectors $\vec{S}$ and $\vec{T}$ are drawn separately. $\vec{S}$ and $\vec{T}$ cannot be added without shifting them so that they touch. The most useful way to make this shift is so that the painted "tip" of one touches the blunt "tail" of the other.

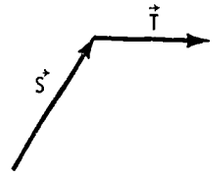
(i) Redraw $\vec{S}$ and $\vec{T}$ with the tip of $\vec{S}$ touching the tail of $\vec{T}$.



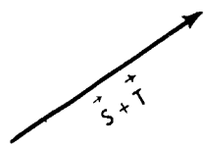
(ii) Draw the vector sum of $\vec{S}$ and $\vec{T}$ on the tip-to-tail drawing.

A9

(i)



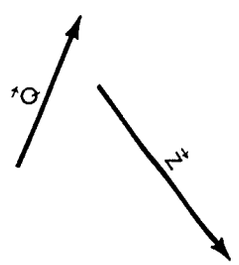
(ii)



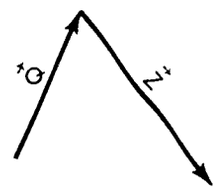
10

(i) Shift the arrow representing the vector $\vec{Z}$ so that its tail is touching the tip of $\vec{Q}$.

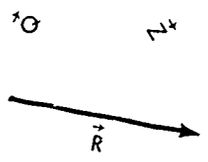
(ii) If $\vec{R} = \vec{Q} + \vec{Z}$, draw an arrow representing $\vec{R}$.



(i)



(ii)

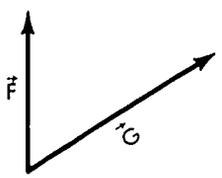


$\vec{H} = \vec{F} + \vec{G}$. Find $\vec{H}$ by adding $\vec{F}$ and $\vec{G}$ with the tip-to-tail method in both of these ways:

(i) shifting $\vec{F}$ to the tip of $\vec{G}$.

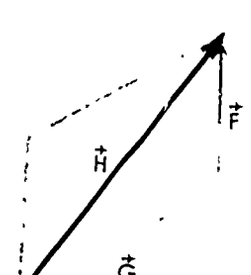
(ii) shifting $\vec{G}$ to the tip of $\vec{F}$.

(iii) Do both procedures give the same result?

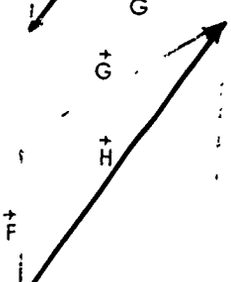


A11

(i)

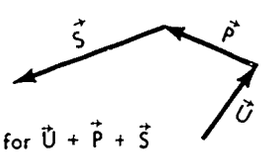


(ii)

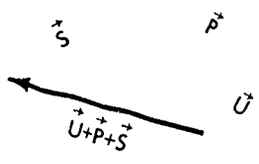


(iii) Yes

The clear advantage of using the tip-to-tail method of graphically adding vectors can be seen when three ar more vectors are to be added. We have already seen this in the example of the city block. The addition is performed by making a "chain" of vectors. Then the sum (or resultant) is found by drawing the arrow from the tail of the first to the head of the last arrow in the chain.



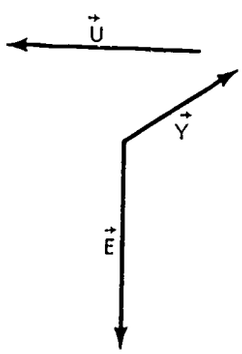
Draw the resultant for $\vec{U} + \vec{P} + \vec{S}$

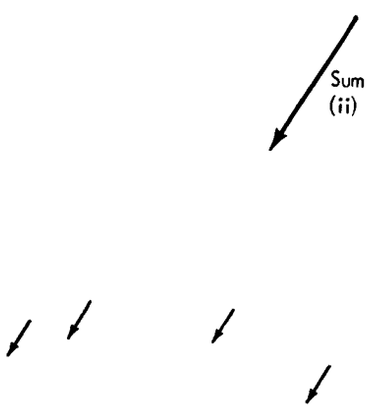

$\vec{S}$
$\vec{P}$
$\vec{U}$
$\vec{U}+\vec{P}+\vec{S}$

13

(i) Redraw $\vec{U}$, $\vec{E}$ and $\vec{Y}$ tip-to-tail.

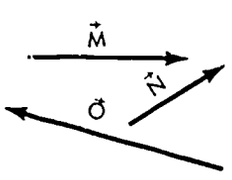
(ii) Draw the vector sum of $\vec{U} + \vec{E} + \vec{Y}$.

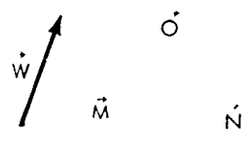



Sum
(ii)

(i) Redraw $\vec{M}$, $\vec{N}$ and $\vec{O}$ tip-to-tail.

(ii) Draw the vector sum $\vec{W}$, where $\vec{M} + \vec{N} + \vec{O} = \vec{W}$.



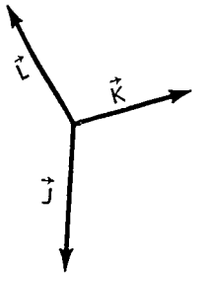


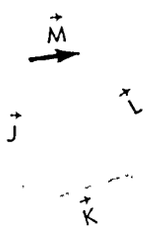
Note:
Any sequence of $\vec{M}$, $\vec{N}$, and $\vec{O}$ will give the same $\vec{W}$.

(i) Redraw the vectors $\vec{J}$, $\vec{K}$ and $\vec{L}$ tip to tail.

(ii) $\vec{J} + \vec{K} + \vec{L} = \vec{M}$. Draw the arrow representing $\vec{M}$.

(iii) Does the order in which you redraw the vectors affect $\vec{M}$?



(i) 

(ii) $\vec{j}$

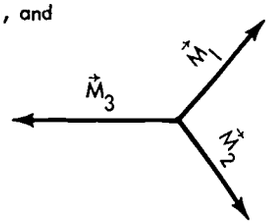
(iii) no

16

Given $\vec{M}_1$, $\vec{M}_2$, $\vec{M}_3$ as shown, and

$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 = \vec{M}_4$

Find $\vec{M}_4$.

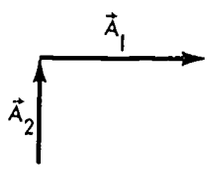


A16

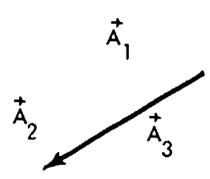
$\vec{M}_4$ is zero

17

If $\vec{A}_1 + \vec{A}_2 + \vec{A}_3 = 0$, and $\vec{A}_1$ and $\vec{A}_2$ are as shown, construct the vector $\vec{A}_3$ that satisfies this equation.



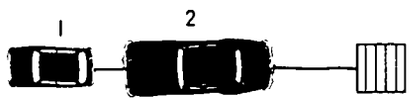
AIV



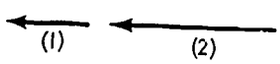
Force is a vector quantity. Each of the cars shown here is exerting a force on the large wooden box.

Draw on the figure the direction of the force each car exerts on the object to which it is hitched.

Suppose the little car (1) pulls with half the force the other car (2) exerts.



Draw on the figure the vector representing the force each car exerts.

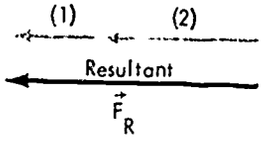

(1)
(2)

Show the sum of the two pulls of the cars, namely the resultant force exerted on the box by both cars pulling together. Assume the pulling forces: $\vec{F}_1$ = 5 units (to the left)

$\vec{F}_2$ = 10 units (to the left

What is the resultant force ($\vec{F}_R$) ?

15 units of force to the left



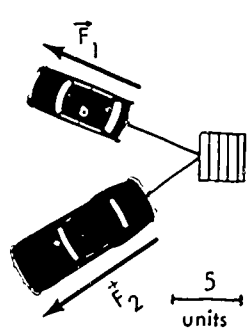
21

Two cars are shown pulling on a wooden box. The pulling force of each car is represented by the vectors $\vec{F}_1$ and $\vec{F}_2$ (note the units).

(i) Construct the vector sum $\vec{F}_R$ of these forces.

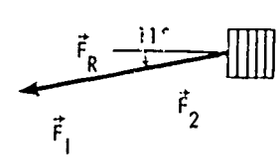
(ii) What is the direction and magnitude of the sum $\vec{F}_R$?

(iii) Write an equation to represent the relation between $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_R$.



A21

(i)



(ii) to the left and a few degrees below horizontal;

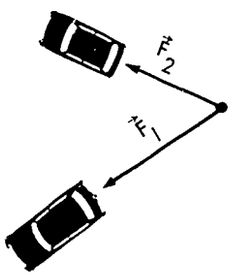
magnitude about 15 units

(iii) $\vec{F}_1 + \vec{F}_2 = \vec{F}_R$

Suppose that two cars were pulling an object, and that each is exerting a force represented by the arrows shown here.

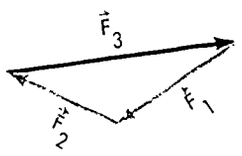
(i) Draw an arrow representing a force vector $\vec{F}_3$ such that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$.

(ii) If $\vec{F}_3$ is the force exerted on the object by a third car, what is the resultant force on the object?



A22

(i)

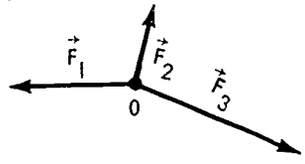


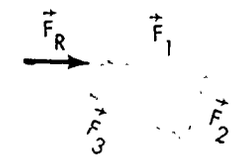
(ii) zero

23

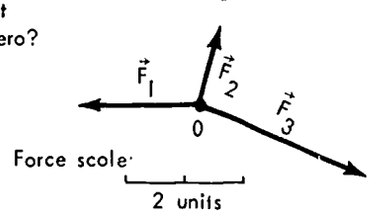
Three forces acting on an object 0 can be represented by arrows as drawn below.

Draw an arrow to represent the resultant force $\vec{F}_R$ on the object.




$\vec{F}_R$
$\vec{F}_1$
$\vec{F}_3$
$\vec{F}_2$

Forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ (from the last frame) are shown acting on object O. You found the resultant force $\vec{F}_R$ by adding these three vectors together "tip-to-toil" in Frame 23. What magnitude should $\vec{F}_1$ have in order to make the resultant force zero?



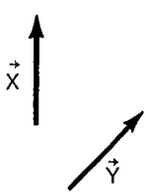
3 units

25

Given are two vectors, $\vec{X}$ and $\vec{Y}$, represented by the arrows drawn here.

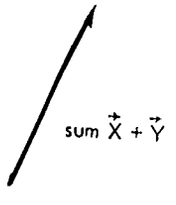
(i) Draw an arrow to represent the vector sum.

(ii) Give its magnitude.



2 units

(i)



(ii) 3.7 units

This ends Vectors 2.

You hove learned how to add two or more vectors together and to draw the resultant vector. Also, given two vectors, you have practiced finding a third vector that would just balance the first two vectors so that the sum of the three was zero.